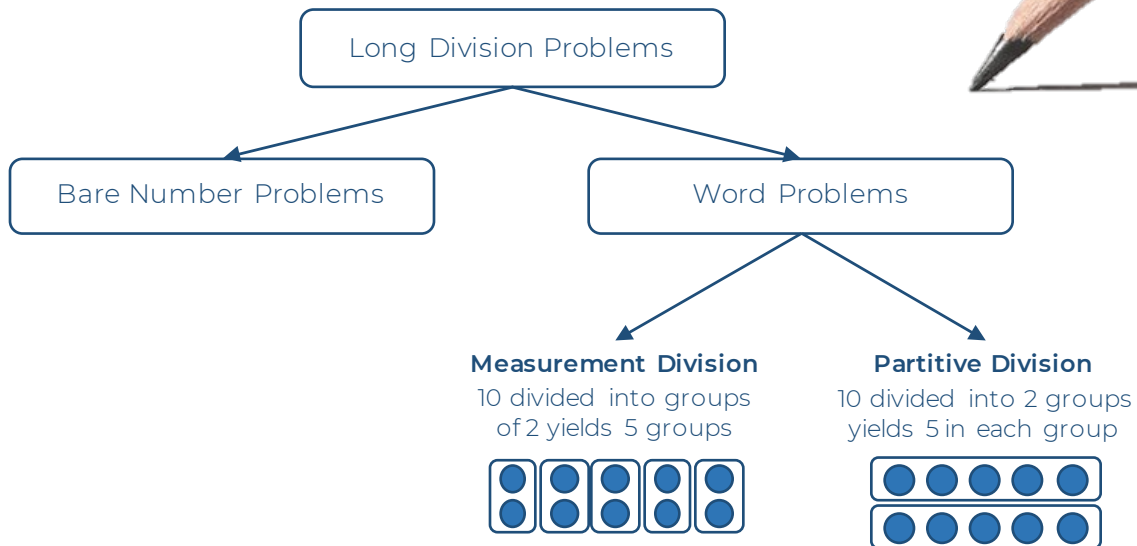


When Pencils Hit Paper

Mastering Long Division



Breaking Down Long Division



4 Types of Remainders

- An extra that cannot be divided
- An extra that must be divided into a fraction
- An irrelevant extra
- A leftover that is central to the problem

Long Division Prerequisites

*taken from Grade 4 Math curriculum

- ✓ Understanding place value up to 10,000
- ✓ Mental 2-digit subtraction
- ✓ 4-digit addition and subtraction
- ✓ 9 times and division tables
- ✓ Mental multiplication and division of any number by 10 and 100

Progressive Mathematization¹²³⁴

Progressive mathematization is an instructional approach to long division, and mathematics in general. It emphasizes mathematical learning as a **process**; students will progress at their **own pace**, using their own **intuitive strategies**. Children **schematize** their strategies and develop **increasingly efficient** strategies through **whole-class discussions** and **teacher guidance**.

Intuitive Student Strategies³

No Chunking (eg., repeated addition/subtraction, doubling of divisor, etc.)

Example: repeated subtraction

$$\begin{array}{r} 15/432 \\ \underline{-15} \quad 1 \\ 417 \\ \underline{-15} \quad 1 \\ 402 \\ \text{etc.} \end{array}$$

Partitioning

$$\begin{array}{l} 400 : 15 = 26 \\ 30 : 15 = 2 \\ 2 : 15 = 1 \end{array}$$

Low-Level Chunking

$$\begin{array}{l} 15+15=30 \\ 30+30=60 \\ 60+60=120 \\ 120+120=240 \\ 240+240=480 \end{array}$$

Low-Level Chunking With a Scheme

$$\begin{array}{r} 15/432 \\ \underline{-60} \quad 4 \\ 372 \\ \underline{-60} \quad 4 \\ 312 \\ \text{etc.} \end{array}$$

High-Level Chunking

$$\begin{array}{l} 10 \times 15 = 150 \\ 150 + 150 = 300 \\ 30 + 30 + 30 + 30 + 15 = 435 \end{array}$$

High-Level Chunking With a Scheme

$$\begin{array}{r} 15/432 \\ \underline{-300} \quad 20 \\ 132 \\ \underline{-120} \quad 8 \\ 12 \quad 28 \end{array}$$



1 - Anghileri, J. (2001). Development of division strategies for year 5 pupils in ten English schools. *British Educational Research Journal*, 27(1), 85-103.

2 - Anghileri, J., Beishuizen, M., & van Putten, K. (2002). From Informal Strategies to Structured Procedures: Mind the Gap! *Educational Studies in Mathematics*, 49(2), 149-170.

3 - van Putten, C. M., van den Brom-Snijders, P. A., & Beishuizen, M. (2005). Progressive mathematization of long division strategies in Dutch primary schools. *Journal for Research in Mathematics Education*, 36(1), 44-73.

4 - Zakaria, E., & Syamaun, M. (2017). The effect of realistic mathematics education approach on students' achievement and attitudes towards mathematics. *Mathematics Education Trends and Research*, 2017(1), 32-40.