

# A Playful Pedagogy Approach to Early Years Mathematics: Focus on Spatial Geometric Reasoning



# Math for Young Children Research Project

- The M4YC project began 7 years ago as a partnership with the Dr. Eric Jackman Institute of Child Study, the Robertson Program, OISE, Trent University and the Ministry of Education
- >2,500 students, > 150 teachers, principals, mathematics coaches
- 8 different school boards
- 5 on-reserve federal schools,
- Ministry personnel



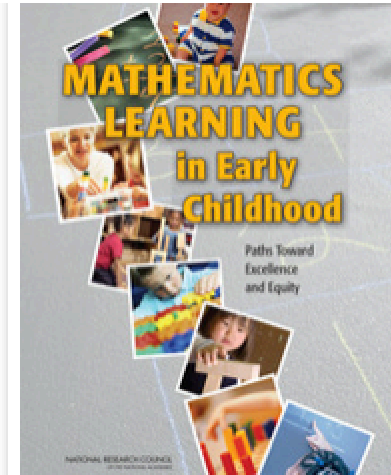
# M4YC 2012-Present

- **OBJECTIVES**
- Broaden knowledge of math teaching and learning in early years focusing on geometry and spatial reasoning
- Focus on under-resourced communities
- Co-create resources for teachers across Ontario
- Conduct research to test new lessons and activities

# Young children's Everyday/Informal Mathematics

- Counting numbers (ordinality)
- Subitizing to 3 or 4 (quantity recognition)
- Recognizing shape or change in shape
- Spatial sense, with emerging awareness of distance, height, location
- **Observed math play for 15 minutes in a classroom setting:**
  - 21% of the time, children explored pattern in shape
  - 13%, children explored magnitude
  - 12%, children explored enumeration
  - (Seo & Ginsburg, 2004)

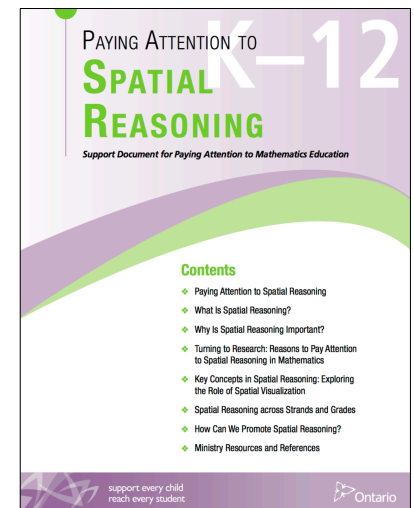
# Early Mathematics for Equity



- Math skills at kindergarten a strong predictor of later academic success, including social studies and reading (Duncan et al, 2007)
- Growth in mathematical ability between kindergarten and first grade is an even stronger predictor of adolescent mathematics achievement.
- “Providing young children with **extensive, high-quality** early mathematics instruction can serve as a sound foundation for later learning and contribute to addressing **long-term systemic inequities** in educational outcomes.” National Research Council

# Spatial Thinking and Geometry for Equity

- Spatial thinking main predictor of, entry, success, creativity and innovation in STEM
- Aesthetic appeal: use of symmetries, beautiful figures and patterns
- Grounds mathematics understanding through body-related experiences
- Highly motivating offering multiple entry points.
- Provides strong and equitable foundation for mathematics learning.



# Spatial Reasoning is Malleable

- Meta analysis of 217 studies over past three decades found an average effect size = 0.47. Effects found for all ages many types of interventions:

Psychological Bulletin

© 2012 American Psychological Association  
0033-2909/12/\$12.00 DOI: 10.1037/a0028446

## The Malleability of Spatial Skills: A Meta-Analysis of Training Studies

David H. Uttal, Nathaniel G. Meadow,  
Elizabeth Tipton, Linda L. Hand, Alison R. Alden,  
and Christopher Warren  
Northwestern University

Nora S. Newcombe  
Temple University

Having good spatial skills strongly predicts achievement and attainment in science, technology, engineering, and mathematics fields (e.g., Shea, Lubinski, & Benbow, 2001; Wai, Lubinski, & Benbow, 2009). Improving spatial skills is therefore of both theoretical and practical importance. To determine whether and to what extent training and experience can improve these skills, we meta-analyzed 217 research studies investigating the magnitude, moderators, durability, and generalizability of training on spatial skills. After eliminating outliers, the average effect size (Hedges's  $g$ ) for training relative to control was 0.47 ( $SE = 0.04$ ). Training effects were stable and were not affected by delays between training and posttesting. Training also transferred to other spatial tasks that were not directly trained. We analyzed the effects of several moderators, including the presence and type of control groups, sex, age, and type of training. Additionally, we included a theoretically motivated typology of spatial skills that emphasizes 2 dimensions: intrinsic versus extrinsic and static versus dynamic (Newcombe & Shipley, in press). Finally, we consider the potential educational and policy implications of directly training spatial skills. Considered together, the results suggest that spatially enriched education could pay substantial dividends in increasing participation in mathematics, science, and engineering.

**Keywords:** spatial skills, training, meta-analysis, transfer, STEM

The nature and extent of malleability are central questions in developmental and educational psychology (Bornstein, 1989). To what extent can experience alter people's abilities? Does the effect of experience change over time? Are there critical or sensitive periods for influencing development? What are the origins and determinants of individual variation in response to environmental input? Spirited debate on these matters is long-standing, and still continues. However, there is renewed interest in malleability in behavioral and neuroscientific research on development (e.g., M. H. Johnson, Munakata, & Gilmore, 2002; National Research Council [NRC], 2000; Stiles, 2008). Similarly, recent economic, educational, and psychological research has focused on the capacity of educational experiences to maximize human potential, reduce inequality (e.g., Duncan et al., 2007; Heckman & Masterov, 2007), and foster competence in a variety of school subjects,

including reading (e.g., Rayner, Foorman, Perfetti, Pesetsky, & Seidenberg, 2001), mathematics (e.g., U.S. Department of Education, 2008), and science and engineering (NRC, 2009).

This article develops this theme further, by focusing on the degree of malleability of a specific class of cognitive abilities: spatial skills. These skills are important for a variety of everyday tasks, including tool use and navigation. They also relate to an important national problem: effective education in the science, technology, engineering, and mathematics (STEM) disciplines. Recent analyses have shown that spatial abilities uniquely predict STEM achievement and attainment. For example, in a long-term longitudinal study, using a nationally representative sample, Wai, Lubinski, and Benbow (2009) showed that spatial ability was a significant predictor of achievement in STEM, even after holding constant possible third variables such as mathematics and verbal skills (see also Humphreys, Lubinski, & Yao, 1993; Shea, Lubinski, & Benbow, 2001).

Efforts to improve STEM achievement by improving spatial skills would thus seem logical. However, the success of this strategy is predicated on the assumption that spatial skills are sufficiently malleable to make training effective and economically feasible. Some investigators have argued that training spatial performance leads only to fleeting improvements, limited to cases in which the trained task and outcome measures are very similar (e.g., Eliot, 1987; Eliot & Fralley, 1976; Maccoby & Jacklin, 1974; Sims & Mayer, 2002). In fact, the NRC (2006) report, *Learning to Think Spatially*, questioned the generality of training effects and concluded that transfer of spatial improvements to untrained skills has not been convincingly demonstrated. The report called for research aimed at determining how to improve spatial performance in a generalizable way (NRC, 2006).

David H. Uttal, Nathaniel G. Meadow, Elizabeth Tipton, Linda L. Hand, Alison R. Alden, and Christopher Warren, Department of Psychology, Northwestern University; Nora S. Newcombe, Department of Psychology, Temple University.

This work was supported by the Spatial Intelligence and Learning Center (National Science Foundation Grant SBE0541957) and by the Institute for Education Sciences (U.S. Department of Education Grant R305H020088). We thank David B. Wilson, Larry Hedges, Loren M. Marulis, and Spyros Konstantopoulos for their help in designing and analyzing the meta-analysis. We also thank Greg Ericksson and Kate O'Doherty for assistance in coding and Kseniya Povod and Kate Bailey for assistance with references and proofreading.

Correspondence concerning this article should be addressed to David H. Uttal, Department of Psychology, Northwestern University, 2029 Sheridan Road, Evanston, IL 60208-2710. E-mail: duttal@northwestern.edu



# Unfortunately....

- Spatial thinking is an underserved area of mathematics instruction
- Despite the push for geometry and spatial thinking to be front and center in early years mathematics curricula
- Geometry very low priority for many teachers in K, 1 and when it is part of the curriculum the focus is on naming and sorting shapes--not on visual and transformational reasoning.
- **Denying children opportunities**
- **Many talents go unnoticed!!**



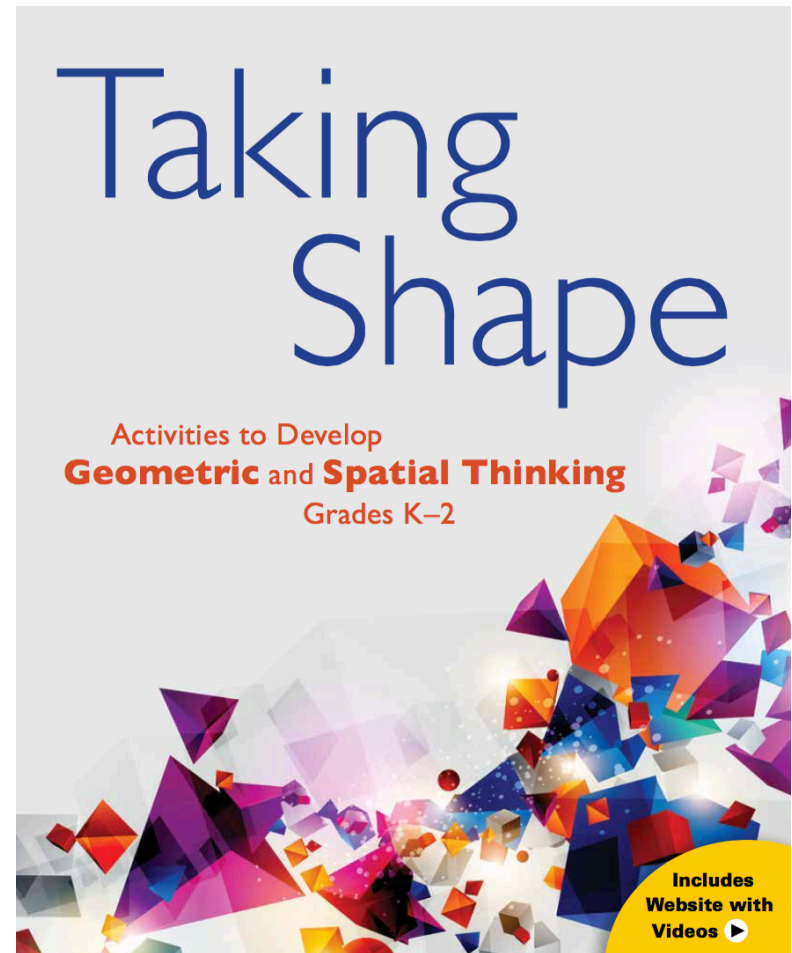
# Our Dynamic Geometry Curriculum

- Symmetry
- Composing, Decomposing and Transforming 2-Dimensional Shape
- Composing, Decomposing and Transforming 3-Dimensional Objects
- Locating, Orienting, Mapping and Coding
- Perspective Taking

-Spatial focus

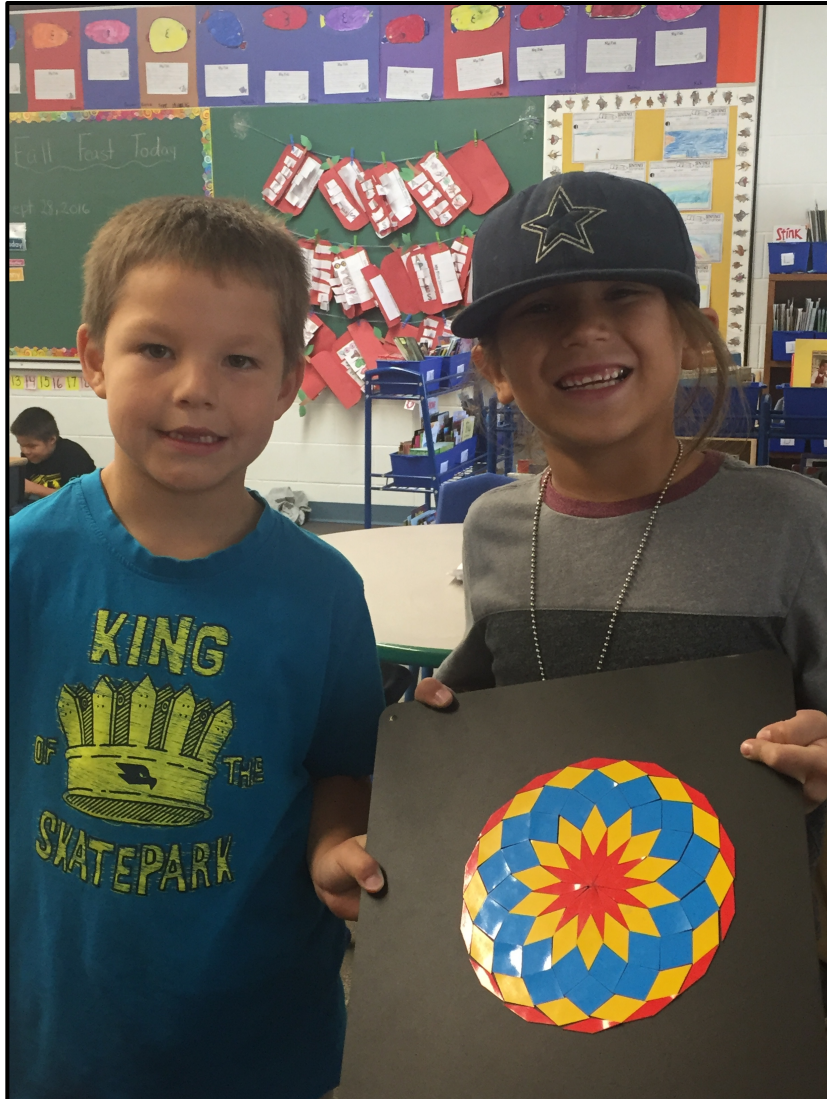
*Spatial visualization*

*Mental rotation*



JOAN MOSS • CATHERINE D. BRUCE • BEV CASWELL • TARA FLYNN • ZACHARY HAWES

# Why focus on Symmetry?



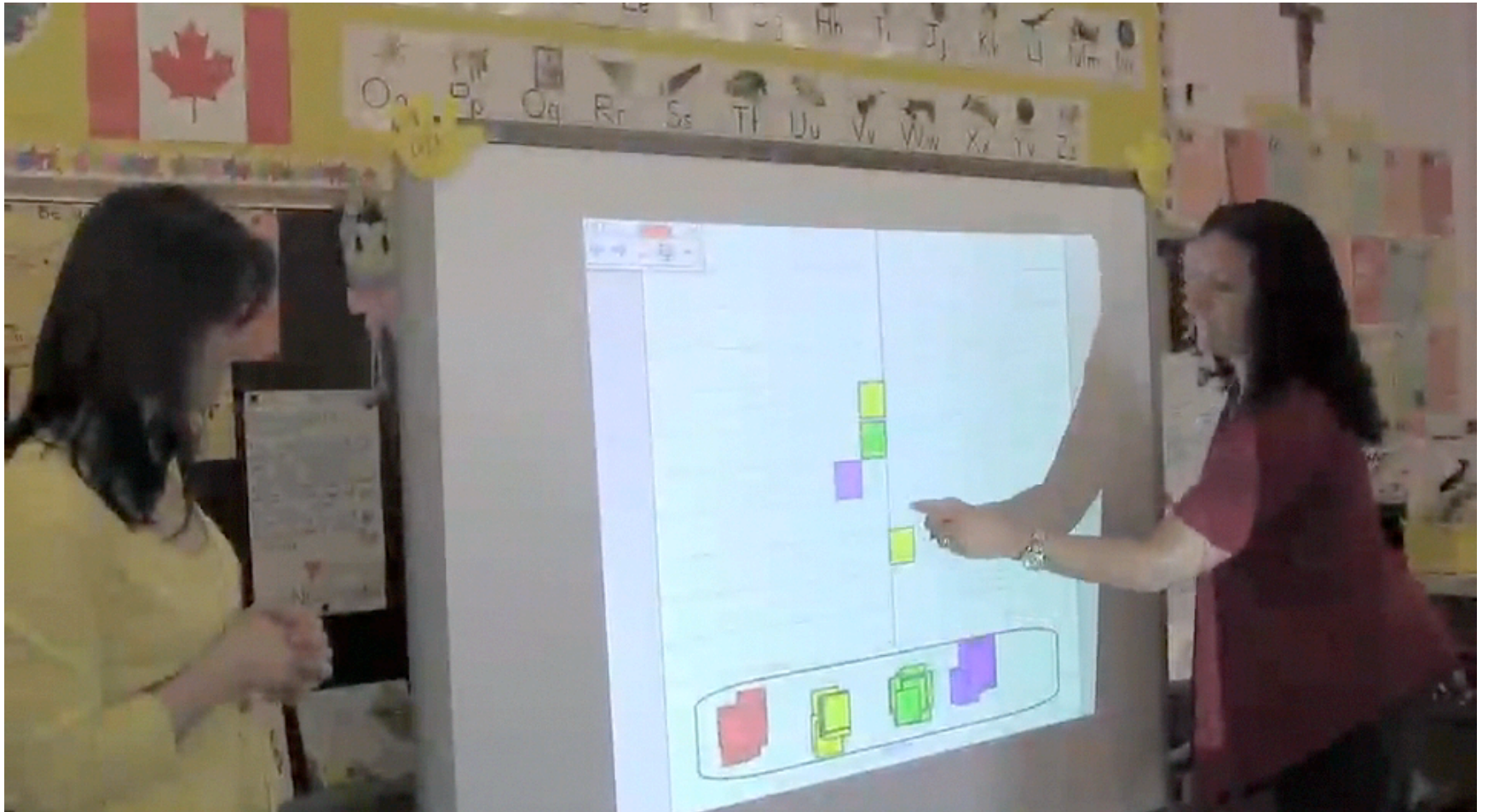
# Symmetry



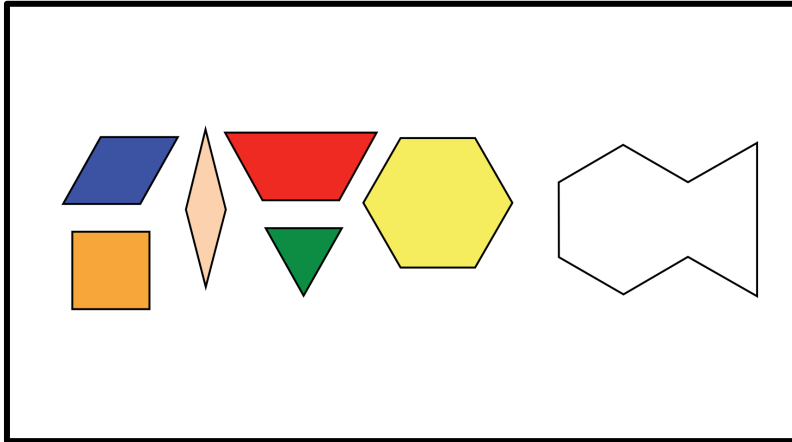
# Introducing Symmetry



# Grid Symmetry Game



# Composing, Decomposing and Transforming 2- and 3- Dimensional Shapes



What are the fewest number of pattern blocks needed to fill the figure on the right. What is the greatest number of blocks needed to fill the figure?

# Composing and Decomposing

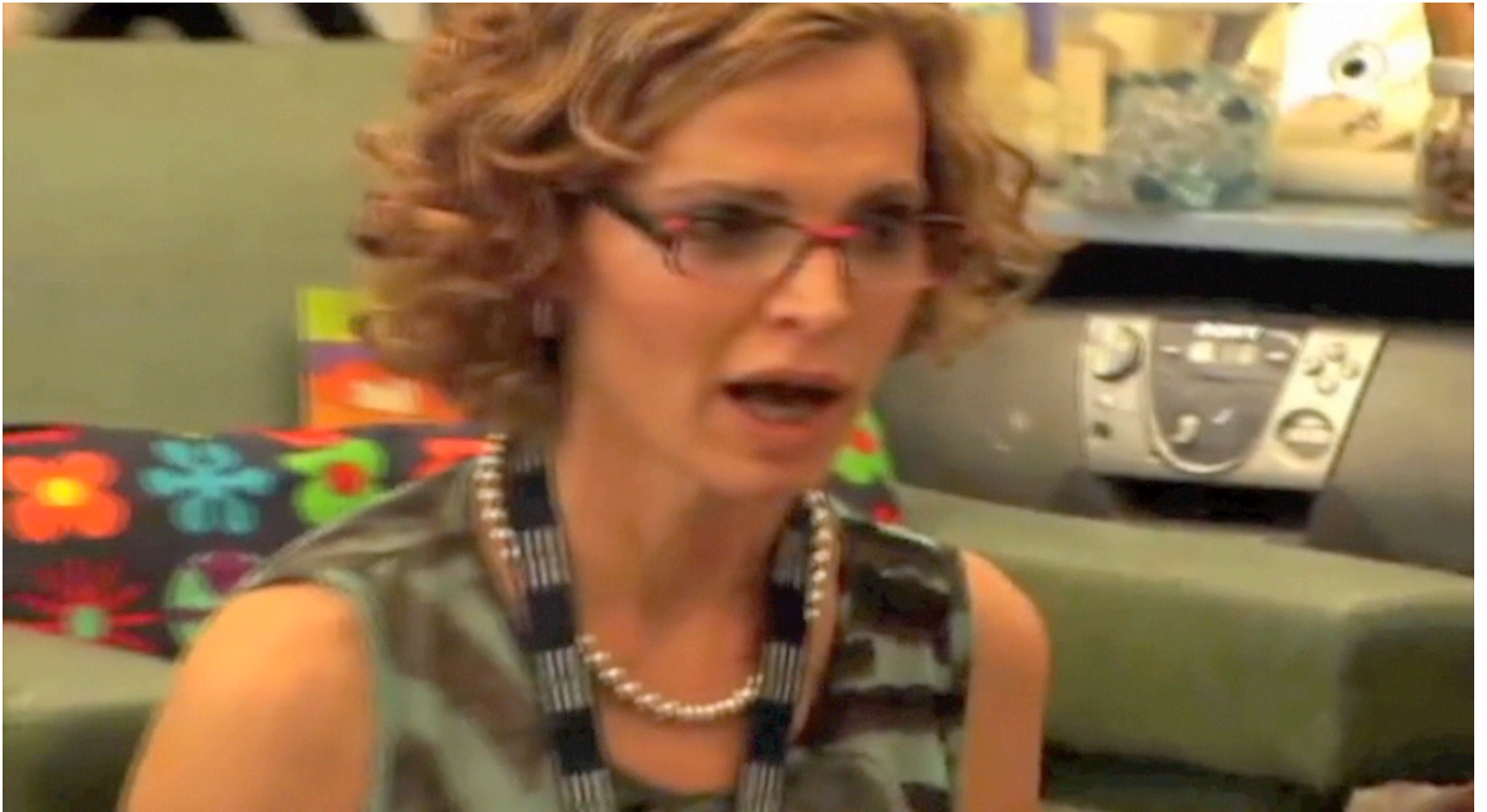


# Spatial Approach to Measurement: Conservation of Area





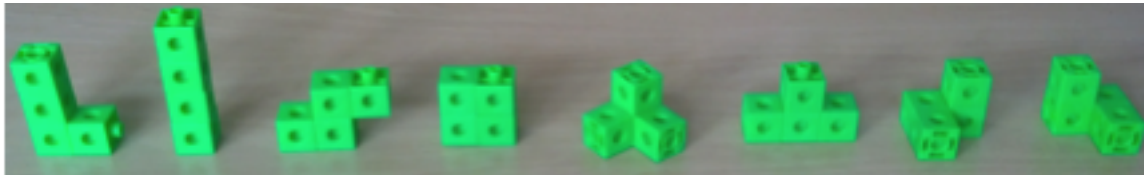
# Garden Tile Lesson



# Congruence/Equivalence: 5-Cube Challenge

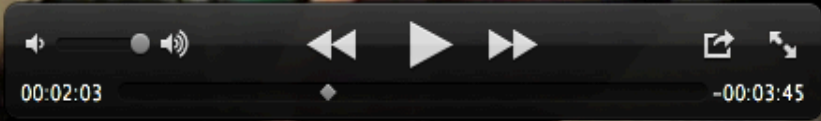
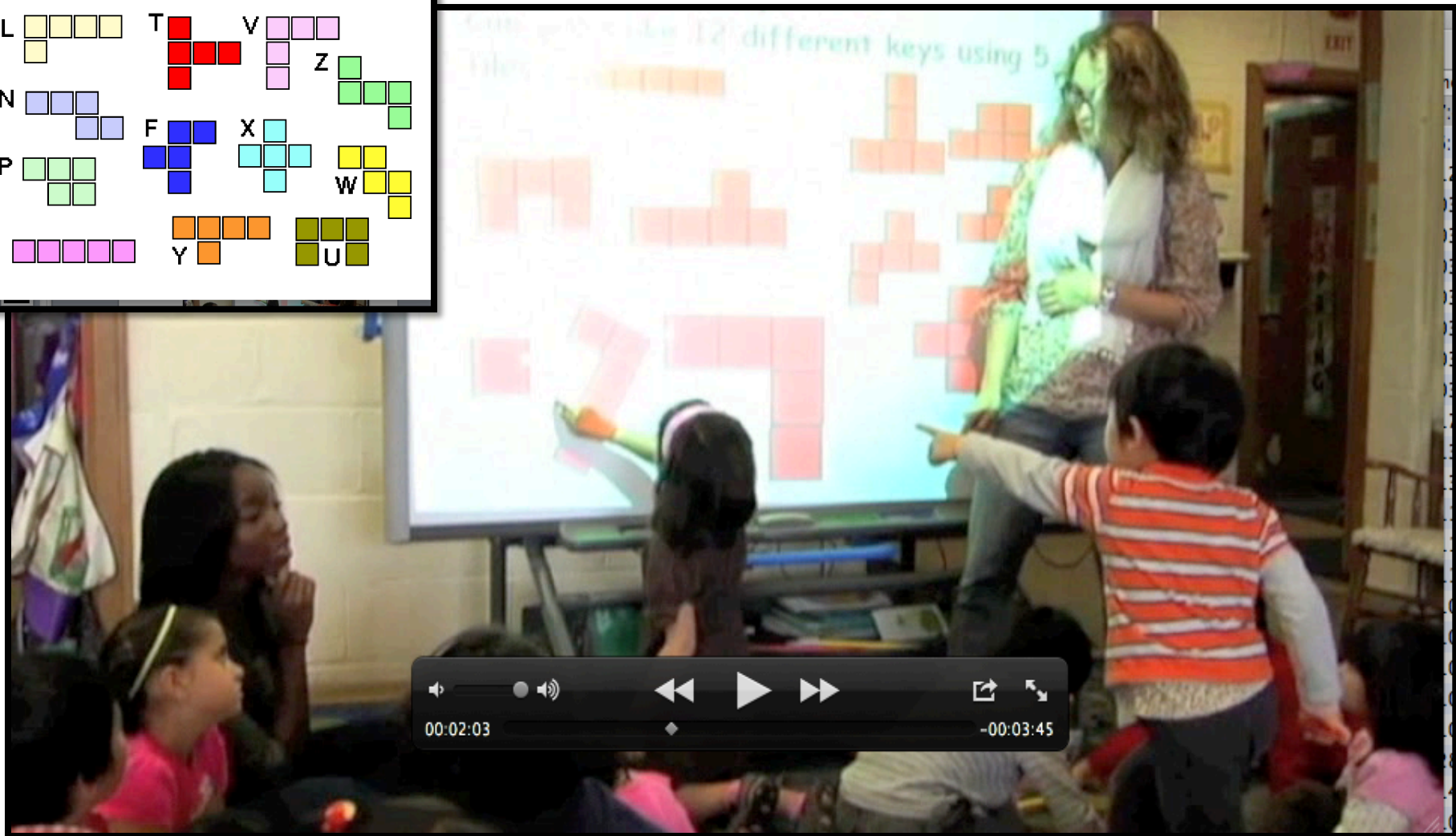
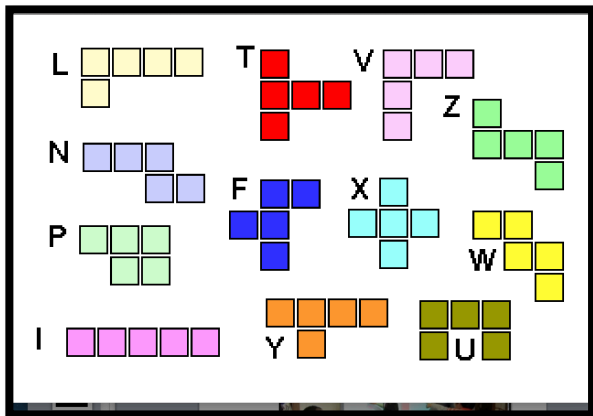


Students introduced to concepts of equivalence, congruence and transformations of 3-dimensional figures in lessons in which they were challenged to find the 28 unique figures that are composed of 5 interlocking cubes.



The 5-Cube Challenge Student at work...

# Pentomino Lesson: "The Magic Keys" Exploring Congruence in 2-dimensions



# Researching in Rainy River District

- 3 schools serving high proportion of FN students
- 2 schools Experimental Group = 38 students, SK - 3
- 1 school active control group = 28 students, SK - 3
- 6 teachers in each group

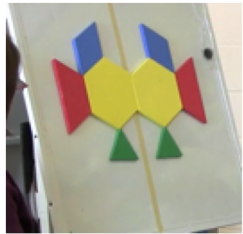


6 days full release both groups; Experimental Group spatial/geometry  
Active control inquiry environmental science + 2 days math number PD

# Activities to Improve Spatial Reasoning



**Fold and Cut Symmetry**



**Big Pattern Block Symmetry**



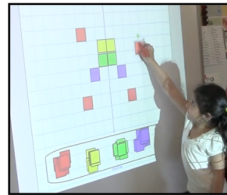
**Cookie Sheet Symmetry**



**Create Symmetry Half**



**Pentomino Symmetry**



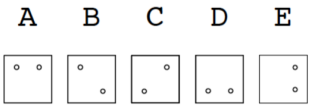
**Grid Symmetry**



**Grid Symmetry Cookie Sheet**



**Alphabet Symmetry**



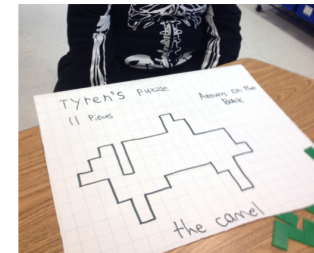
**Folding and Hole Punch**



**Symmetry Concentration**



**Magic Key/Pentomino Lesson**



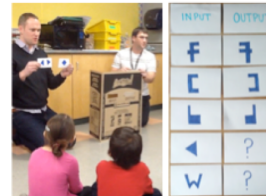
**Create Pentomino Puzzle**



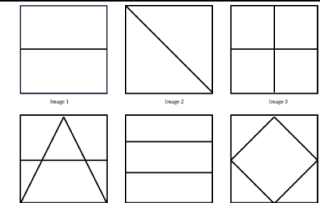
**Pattern Block Designs**



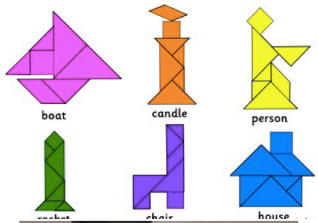
**Hexagon Card Game**



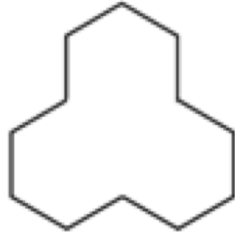
**Shape Transformer**



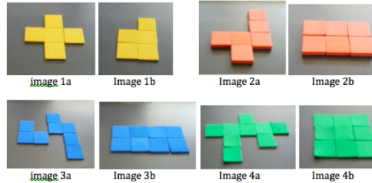
**Can You Draw This?**



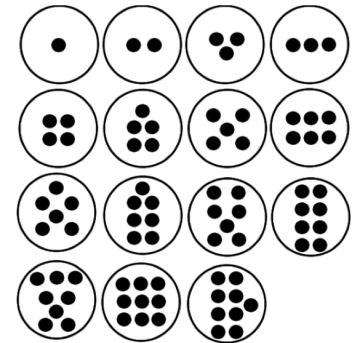
**Tangram Shapes**



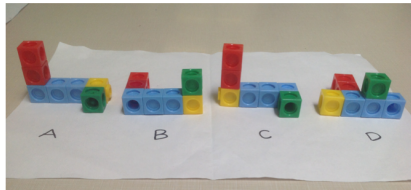
**Can You Cover This?**



**Shape Mover**



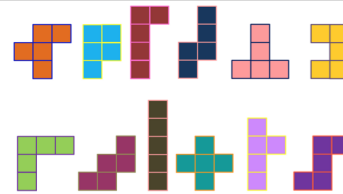
**Dot Plates: Can You Count It**



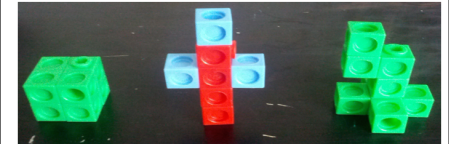
**Build it in Your Mind**



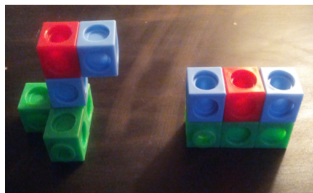
**Building with Frameworks**



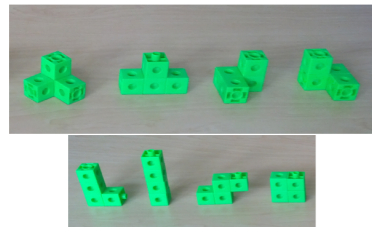
**Create an Open Box**



**See It Build It**



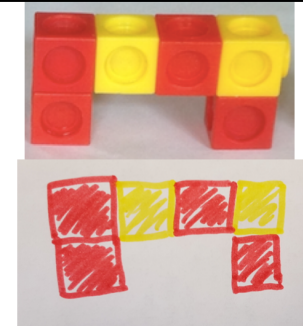
**Building with Rules**



**Four Cube Challenge**



**Five Cube Challenge**

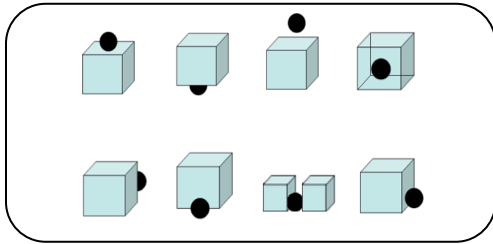


**Build It Draw It**

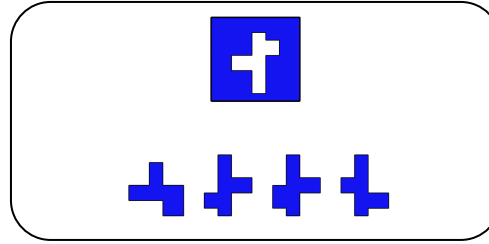
Average 45 hours over 7 months mostly on quick image activities

# Results

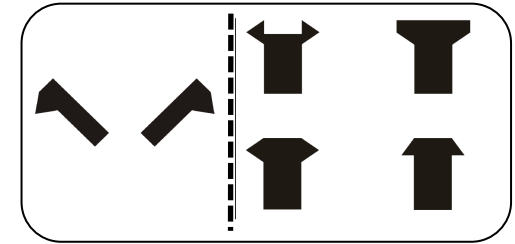
Hawes, Z., Moss, J., Caswell, B., Naqvi S. & MacKinnon (2017) Enhancing Children's Spatial and Numerical Skills through a Dynamic Spatial Approach to Early Geometry Instruction: Effects of a 32-Week Intervention, *Cognition and Instruction*, 35:3, 236-264,



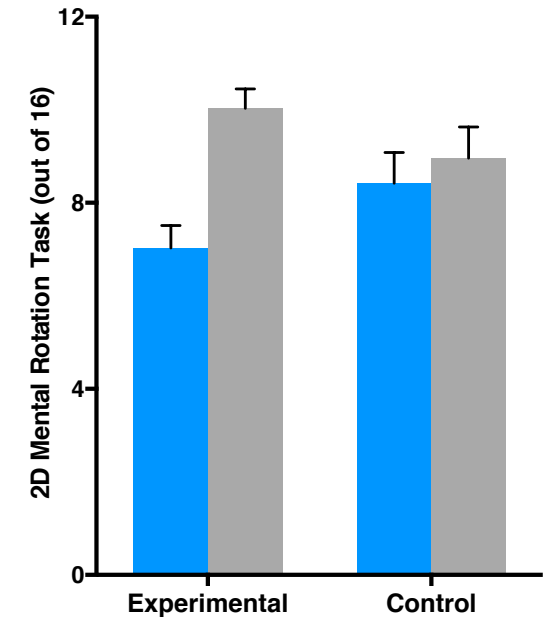
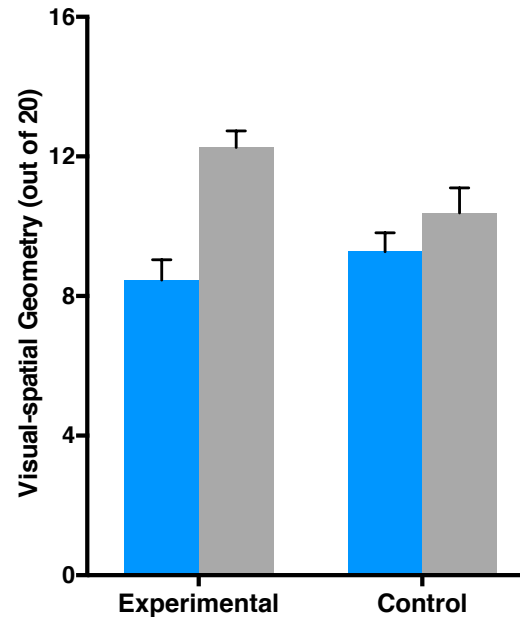
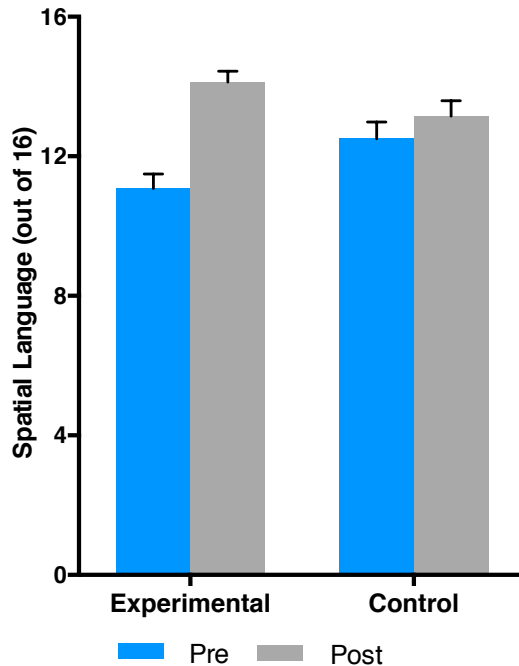
\*



\*

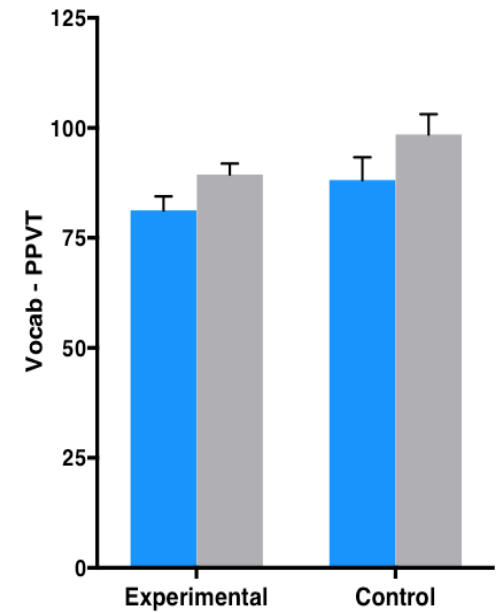
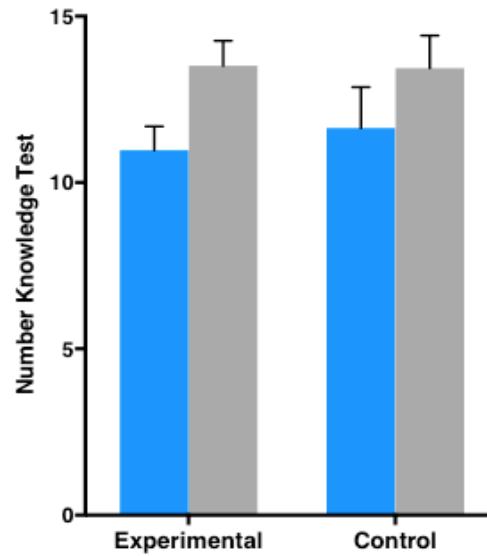
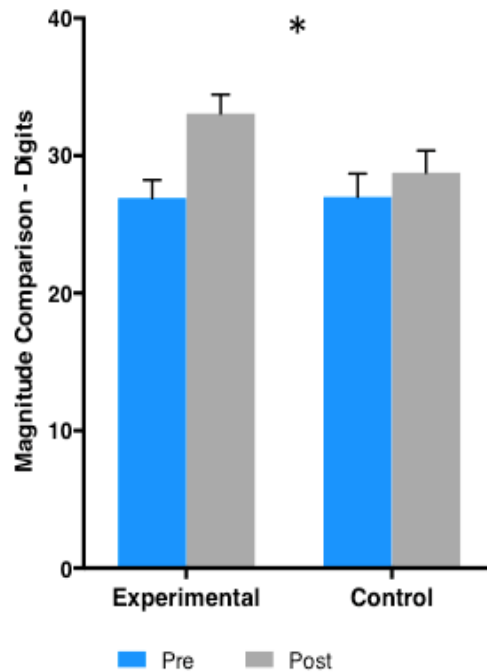
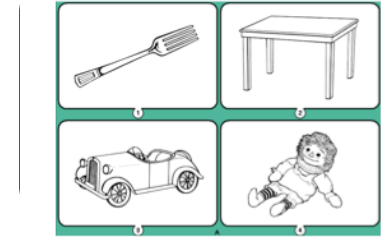


\*



\* = sig. group x time interaction,  $p < .05$

# Results





# Playful Pedagogy in M4YC

- Playful/engaging contexts
  - Narratives
  - Fantasy
  - Game-like
- Important rigorous math
- Child-centered
- Authentic exploration
- Lots of choice
- Inquiry and discovery
- Problem oriented
- Action orientation
- Circle formation
- Visibility of learning
- “Engaging the collective”



# Playful Pedagogy in M4YC

- Structuring of a learning context and materials around a particular set of developmental mathematical goals.
- Leading responsive, inquiry-based learning activities that include exploration with specific objects/materials that stimulate children's curiosity, engagement, and sense making.



*SK exploration of Proportional Reasoning*

# Thank You



# Researching Year 2

Carried out another intervention study, but made some changes:

- New schools (same board)
- More teachers ( $n = 10$ )
- More measures, including standardized Canadian normed math assessments
- More child participants (**experimental  $n = 85$** )

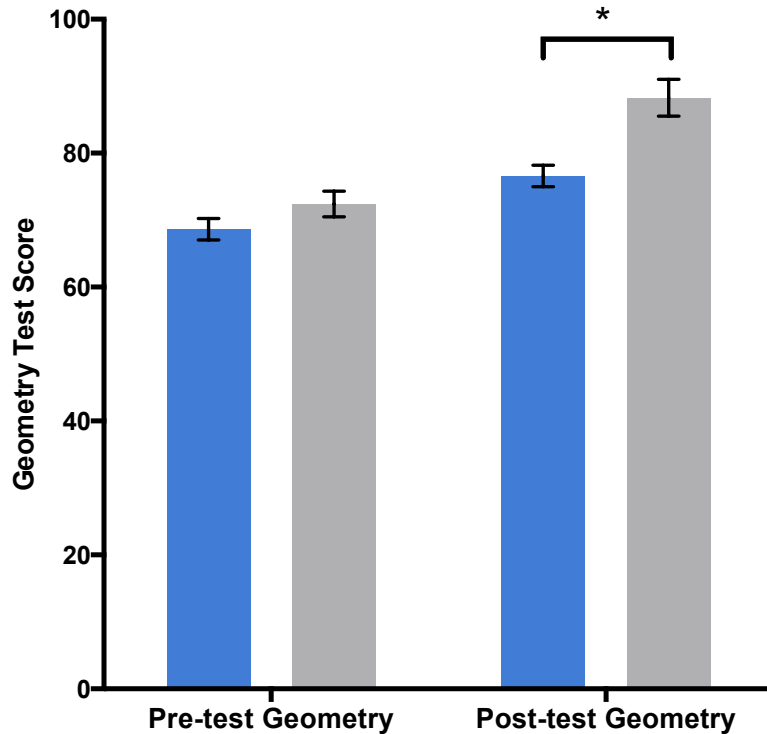


# Year 2: Results Geometry and Numeration

## Key Math

Control  
Experimental

Improvements in Geometry Performance



Improvements in Numeration Performance

